$$\begin{array}{cccc} u(e_1) & u(e_2) & u(e_3) & \\ \left(\begin{matrix} a \\ 0 \\ 0 \end{matrix}\right. & \begin{matrix} b \\ c \\ 0 \end{matrix} & \left.\begin{matrix} d \\ e \\ f \end{matrix}\right) & \begin{matrix} e_1 \\ e_2 \\ e_3 \end{matrix} \end{array}$$

$$\begin{array}{cccc} u(e_3) & u(e_2) & u(e_1) & \\ \left(\begin{matrix} f \\ e \\ d \end{matrix}\right. & \begin{matrix} 0 \\ c \\ b \end{matrix} & \left.\begin{matrix} 0 \\ 0 \\ a \end{matrix}\right) & \begin{matrix} e_3 \\ e_2 \\ e_1 \end{matrix} \end{array}$$

Rmq 5

(1)

$$\begin{array}{cc} & \begin{matrix} u(e_1) & \cdots & u(e_i) & \cdots & u(e_n) \end{matrix} \\ M_{\mathcal{B}}(u) = & \begin{pmatrix} \ddots & & \vdots & & \\ & \ddots & \vdots & & \\ & & \cdot & \cdots & \\ & & 0 & \ddots & \\ & & \vdots & & \ddots \\ & & 0 & & \end{pmatrix} \begin{matrix} e_1 \\ \vdots \\ e_i \\ \vdots \\ e_n \\ \end{matrix} \end{array}$$

$M_{\mathcal{B}}(u)$ triang. sup. $\Leftrightarrow \forall i,\ u(e_i) \in \mathrm{Vect}(e_1, \dots, e_i)$

$M_{\mathcal{B}}(u)$ diagonale $\Leftrightarrow \forall i,\ u(e_i) \in \mathrm{Vect}(e_i)$

(3) $\longrightarrow$ exercice 8

χ_u scindé $\Leftrightarrow$ u trigonalisable

$+ \dim E_\lambda(u) = m(\lambda) \ \forall \lambda \in Sp(u)$

$\Leftrightarrow$ diagonalisable

Dimension 3

Imaginons avoir 2 $\vec{v}p$ e_1, e_2

associés à λ_1, λ_2 :

$$\underset{(e_1, e_2, \dots)}{M(u)} = \begin{pmatrix} \lambda_1 & 0 & a \\ 0 & \lambda_2 & b \\ 0 & 0 & c \end{pmatrix}$$

choisi arbitrairement en veillant à ce que $(e_1, e_2, \dots)$ soit libre.

Calculs des puissances

$$M = P \cdot D \cdot P^{-1} \ ; \quad M^k = P \cdot D^k P^{-1}$$

preuve par récurrence

$$M^2 = P \cdot D \cdot \underbrace{P^{-1} P}_{I_n} D P^{-1} = P D^2 P^{-1}$$

$$u_{n+3} = -3u_{n+2} + u_{n+1} + 3u_n \quad \in \mathbb{R}$$

$$U_n = \begin{pmatrix} u_n \\ u_{n+1} \\ u_{n+2} \end{pmatrix}. \qquad \boxed{U_{n+1} = AU_n} \in \mathcal{M}_{3,1}(\mathbb{R})$$

$$\begin{pmatrix} u_n \\ u_{n+1} \\ u_{n+2} \end{pmatrix} = U_n$$

$$U_{n+1} = \begin{pmatrix} u_{n+1} \\ u_{n+2} \\ u_{n+3} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & 1 & -3 \end{pmatrix}$$

$$U_{n+1} = AU_n \xrightarrow[\text{récurrence}]{} U_n = A^n U_0$$

$$\begin{pmatrix} u_0 \\ u_1 \\ u_2 \end{pmatrix}$$

$$U_n = \begin{pmatrix} u_n \\ u_{n+1} \\ u_{n+2} \end{pmatrix} \times$$

$$(r-1)(r-2) = 0 \qquad \boxed{u_{n+2} = 3u_{n+1} + 2u_n}$$

$$r^2 - 3r + 2 \qquad \hookrightarrow u_n = \underline{C \cdot 1^n} + D \cdot 2^n$$

$$A = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \longrightarrow \chi_A(\lambda) = \lambda^2 - 3\lambda + 2$$